Perturbative Odderon in the Dipole Model

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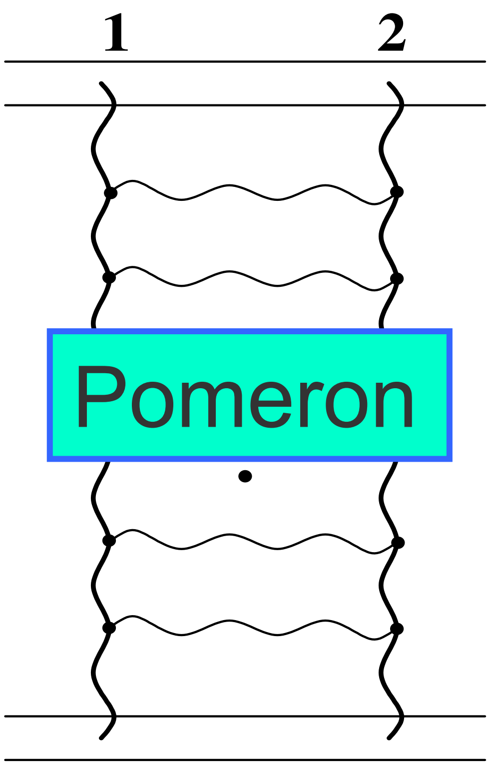
based on work done in collaboration with
L. Szymanowski and S. Wallon, hep-ph/0309281

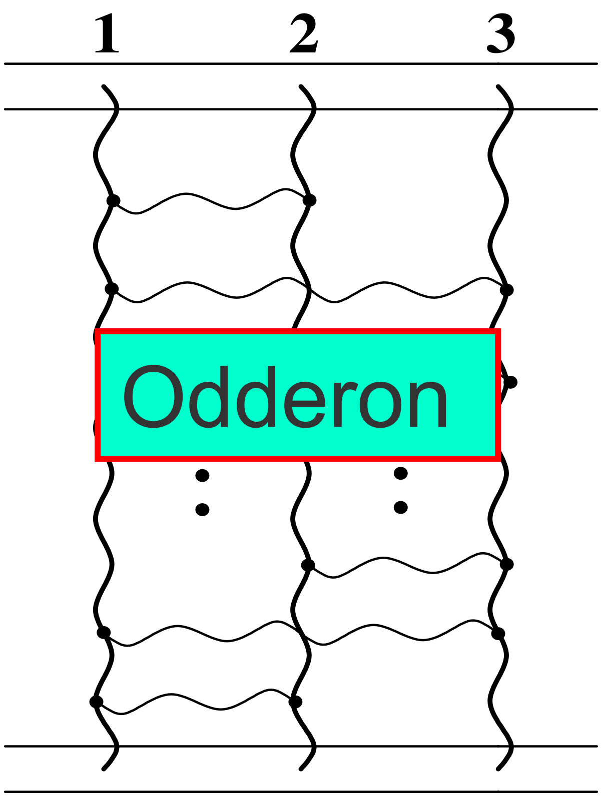
The BKP Equation

The BFKL equation describes evolution of the 2-reggeon system shown below (a C-even exchange)



$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$

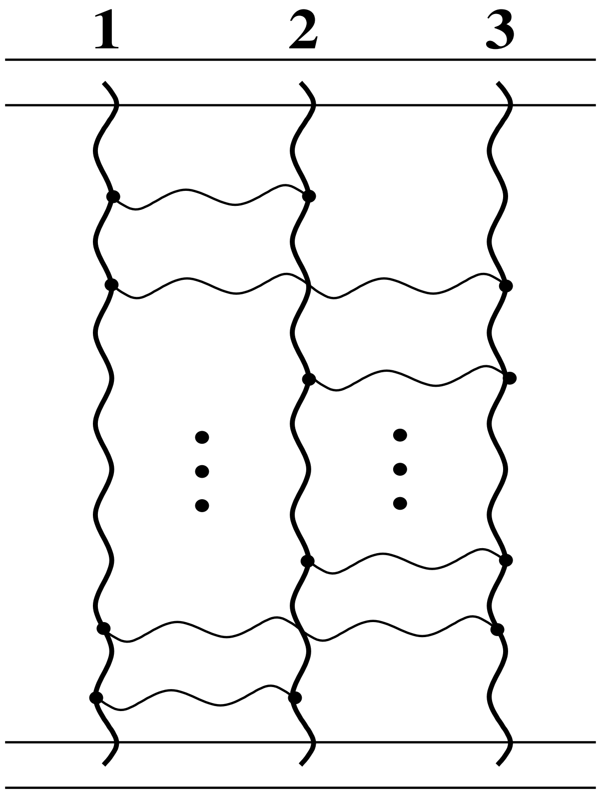
The BKP equation describes evolution of any n-reggeon system. For 3-reggeons it gives a C-odd amplitude



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

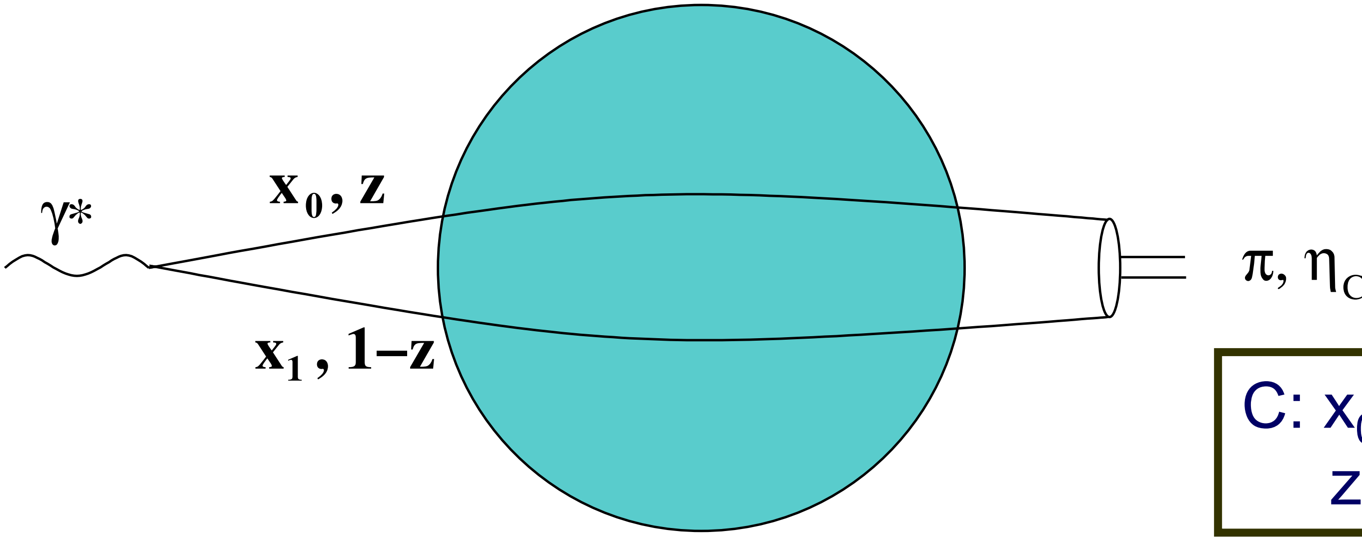
The Odderon Problem

While a complete set of eigenfunctions for the BFKL equation has been found long time ago, it has not been found for the BKP equation with 3 reggeons. Instead there are several solutions of the 3-reggeon BKP equation on the market, some giving different leading odderon intercepts. Since a complete set of eigenfunctions has not been found, one can not prove that any particular solution is dominant at high energy, since there is always a chance to find more.



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

Dipole Approach



C: $x_0 \leftrightarrow x_1$
$z \leftrightarrow 1\text{-}z$

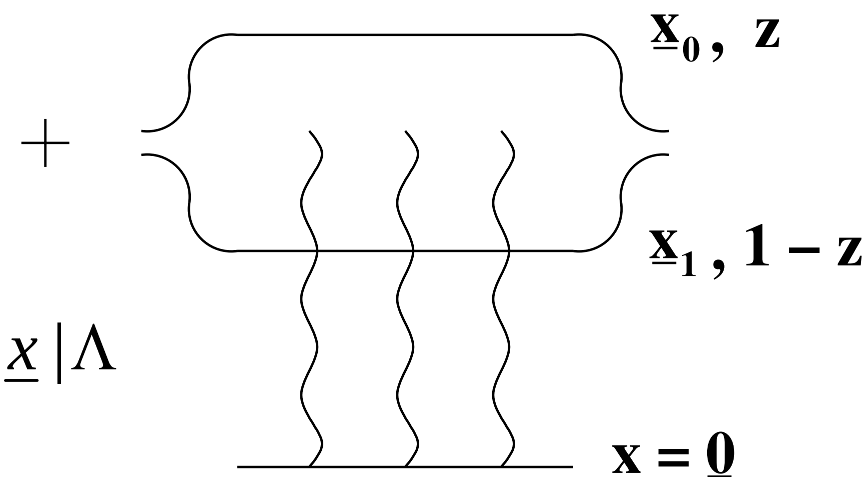
Our approach is somewhat pragmatic: for us an odderon exchange is any diffractive process that changes the C-parity of the quark-antiquark light cone wave function (e.g. a C-odd virtual photon can become a C-even pion, etc.). Corresponding diffractive dipole amplitude has to satisfy the following conditon:

$$\boxed{O(\underline{x}_0, \underline{x}_1; z, 1-z) = -O(\underline{x}_1, \underline{x}_0; 1-z, z)}$$

Lowest Order

At the lowest order the C-odd odderon amplitude is given by the three-gluon exchange in the color-singlet state:

$$\int \frac{d^2k}{k^2} e^{i\underline{k}\cdot\underline{x}} \sim \ln|\underline{x}|\Lambda$$



$$O_0(\underline{x}_0, \underline{x}_1) = c_0\, \alpha_S^3 \ln^3 \frac{x_0}{x_1}$$

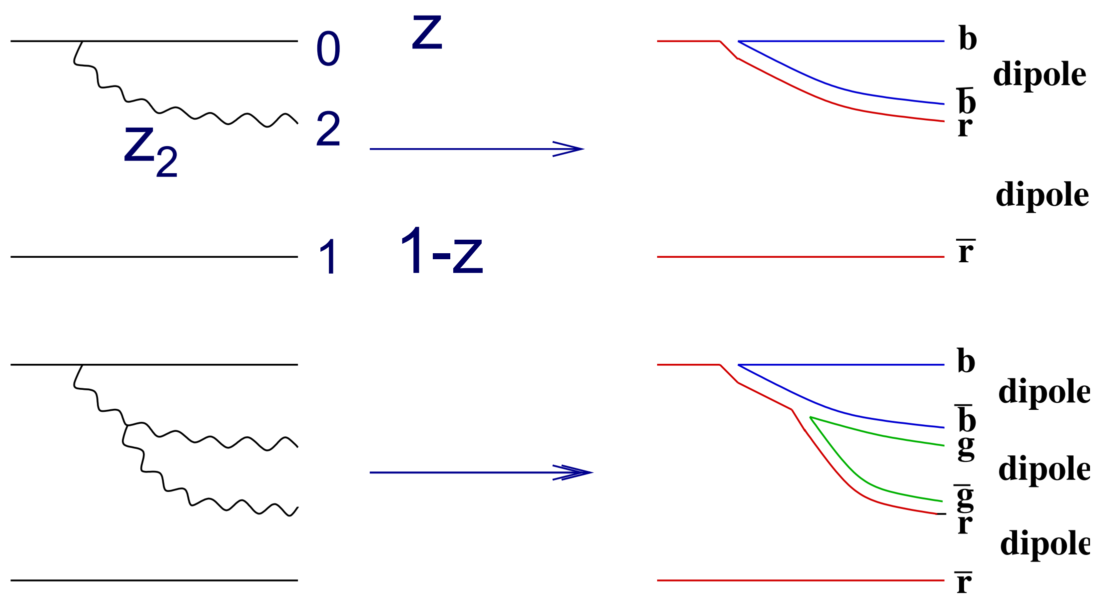
with c_0 the color factor. The target is located at $\underline{0}$ transverse coordinate.

The above amplitude obviously changes sign under $x_0 \leftrightarrow x_1$!
(All eikonal amplitudes are even under $z \leftrightarrow 1$-z interchange.)

Mueller's Dipole Model

To include the quantum evolution in a dipole amplitude one has to use the approach developed by A. H. Mueller in ’93-’94. The goal is to resum leading logs of energy, $\alpha \log s$, just like for the BFKL equation.

Emission of a small-x gluon taken in the large-N_C limit would split the original color dipole in two:

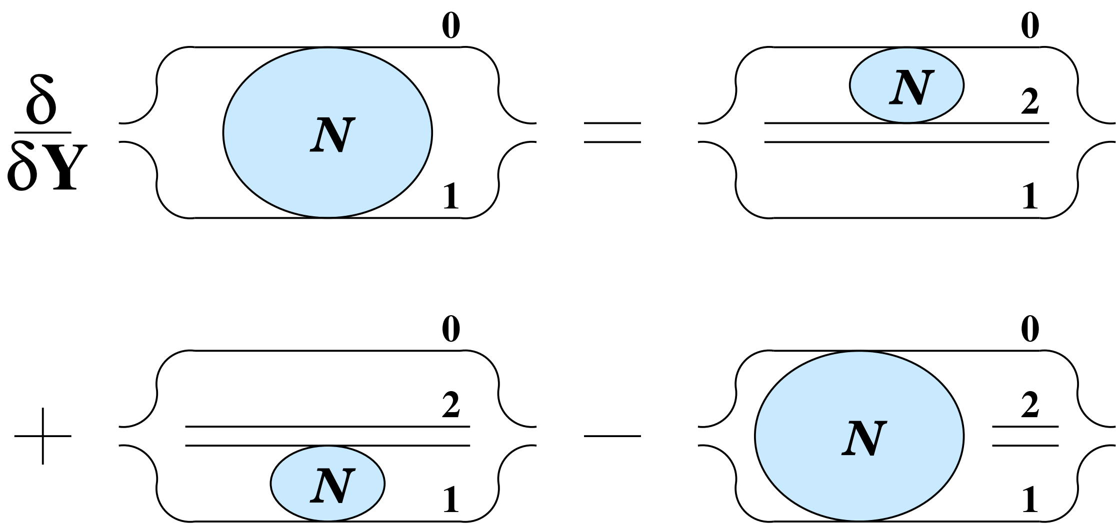


Emission of a single gluon with longitudinal momentum fraction z_2 and transverse coordinate x_2 would bring in a factor

$$\frac{\alpha_S N_C}{2\pi^2} \int_{z_{init}}^{\min\{z,1-z\}} \frac{dz_2}{z_2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2}$$

Mueller's Dipole Model

For the dipole scattering amplitude N one can write down an equation resumming all successive gluon emissions:



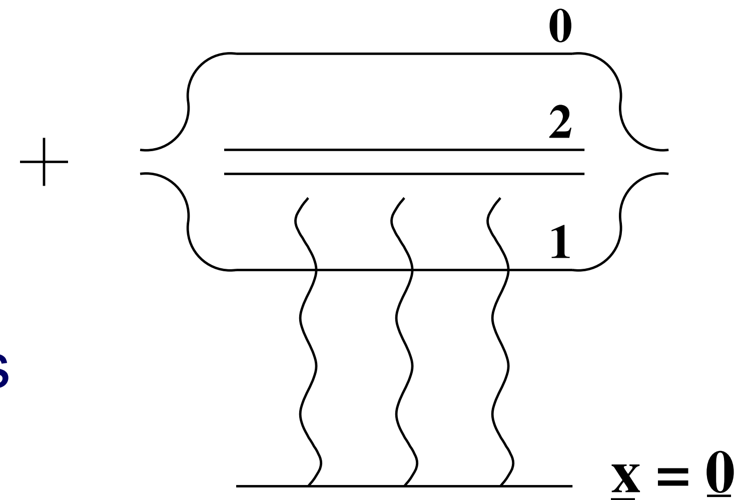
$$\frac{\partial N(\underline{x}_0,\underline{x}_1,Y)}{\partial Y}=\frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\left[N(\underline{x}_0,\underline{x}_2,Y)+N(\underline{x}_2,\underline{x}_1,Y)-N(\underline{x}_0,\underline{x}_1,Y)\right]$$

A. H. Mueller, '93

This equation is equivalent to the BFKL equation (more later)!

One Rung of Odderon Evolution

Let us include one rung of the small-x evolution in the framework of A. H. Mueller's dipole model. We are working in the large N_C limit, so the graphs are planar and the gluon is given by double line:



One rung of evolution gives

$$\frac{\alpha_S N_C}{2\pi^2}\int_{z_{init}}^{\min\{z,1-z\}}\frac{dz_2}{z_2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}[O_0(\underline{x}_0,\underline{x}_2)+O_0(\underline{x}_2,\underline{x}_1)-O_0(\underline{x}_0,\underline{x}_1)]$$

Substituting the amplitude O_0 from the previous slide we get

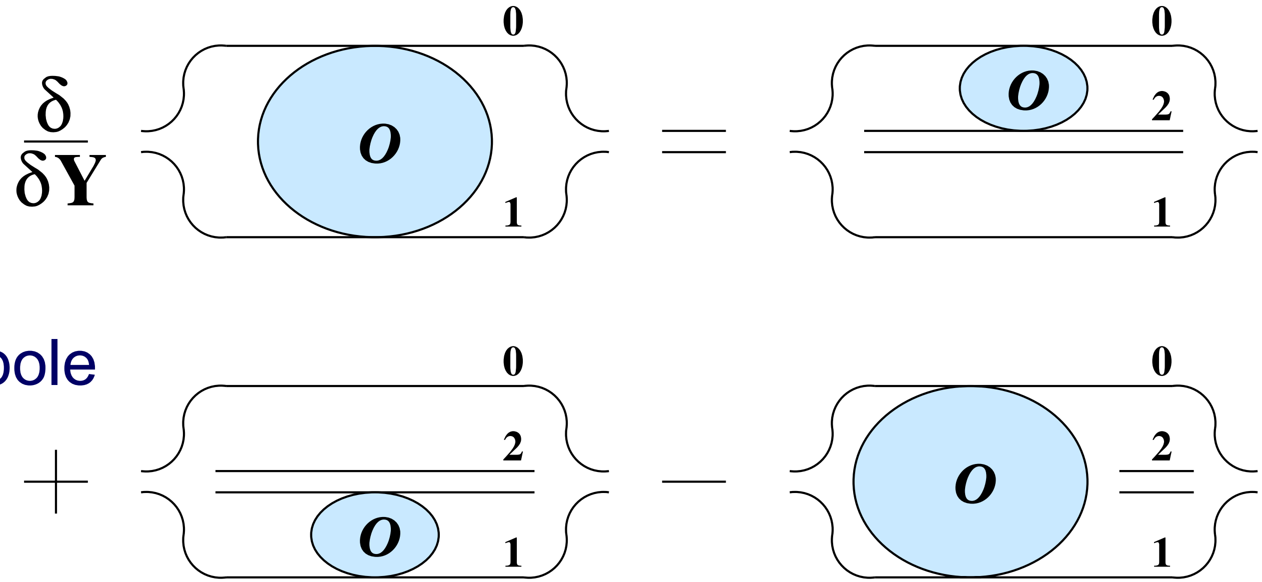
$$3\frac{\alpha_S N_C}{2\pi^2}c_0\,\alpha_S^3\ln\frac{x_0}{x_1}\ln\frac{\min\{z,1-z\}}{z_{init}}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\ln\frac{x_0}{x_2}\ln\frac{x_1}{x_2}$$

which is non-zero and C-odd (changes sign under $x_0 \leftrightarrow x_1$).

⇨ "C-oddness" survives dipole evolution!

Odderon Evolution Equation

Now we can easily write down an evolution equation for the odderon amplitude in the dipole model.



$$\frac{\partial O(\underline{x}_0,\underline{x}_1,Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\left[O(\underline{x}_0,\underline{x}_2,Y)+O(\underline{x}_2,\underline{x}_1,Y)-O(\underline{x}_0,\underline{x}_1,Y)\right]$$

This is the same evolution equation as for the BFKL Pomeron in the dipole model! How is the Odderon equation different from the Pomeron equation?

Odderon Evolution Equation

$$\frac{\partial O(\underline{x}_0, \underline{x}_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} [O(\underline{x}_0, \underline{x}_2, Y) + O(\underline{x}_2, \underline{x}_1, Y) - O(\underline{x}_0, \underline{x}_1, Y)]$$

⇨ Odderon and Pomeron are described by the <u>same evolution equation</u> in Mueller's dipole model!

⇨ The only difference is in the <u>initial conditions</u>: Odderon has a C-odd initial condition,

$$O(\underline{x}_0, \underline{x}_1, Y = 0) = O_0(\underline{x}_0, \underline{x}_1) = c_0 \, \alpha_S^3 \ln^3 \frac{x_0}{x_1}$$

while the Pomeron has a C-even initial condition given by 2-gluon exchange:

$$N(\underline{x}_0, \underline{x}_1, Y = 0) \sim \alpha_S^2 \ln^2 \frac{x_0}{x_1}$$

⇨ Initial Conditions single out either the Odderon or the Pomeron contributions depending on their C-parity!

Solution of the Dipole Odderon Equation

Since the Odderon evolution equation is equivalent to the BFKL Pomeron equation the solution can be easily constructed. First one shows that the eigenfunctions of conformal algebra Casimirs

$$E^{n,\nu}(\rho_0,\rho_1) = \left(\frac{\rho_{01}}{\rho_0\,\rho_1}\right)^{\frac{1+n}{2}+i\nu} \left(\frac{\rho^*{}_{01}}{\rho^*{}_0\,\rho^*{}_1}\right)^{\frac{1-n}{2}+i\nu}$$

are also the eigenfunctions of the equation's kernel

$$\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\;x_{12}^2}[E^{n,\nu}(\underline{x}_0,\underline{x}_2)+E^{n,\nu}(\underline{x}_2,\underline{x}_1)-E^{n,\nu}(\underline{x}_0,\underline{x}_1)] = 4\pi\,\chi(n,\nu)\,E^{n,\nu}(\underline{x}_0,\underline{x}_1)$$

with the eigenvalues being the same as for the BFKL equation

$$\chi(n,\nu) = \psi(1) - \frac{1}{2}\psi\left(\frac{1+|n|}{2}+i\nu\right) - \frac{1}{2}\psi\left(\frac{1+|n|}{2}-i\nu\right)$$

Solution of the Odderon Equation

Since $$E^{n,\nu}(\rho_0,\rho_1)=(-1)^n E^{n,\nu}(\rho_1,\rho_0)$$

the C-odd initial condition would pick only the eigenfunctions

$E^{n,\nu}(\rho_0,\rho_1)$ with odd n. These are the eigenfunctions of the odderon evolution equation.

The eigenvalues would then be given by $\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)$ with odd n.

The odderon solution would then look like

$$\int_{-\infty}^{\infty} d\nu \sum_{odd\, n} \exp\left[\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)Y\right]E^{n,\nu}(\rho_0,\rho_1)$$

Solution of the Odderon Equation

To find the leading high energy term in

$$\int_{-\infty}^{\infty} d\nu \sum_{odd\, n} \exp\left[\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)Y\right] E^{n,\nu}(\rho_0,\rho_1)$$

one notes that the saddle point of ν-integrals is given by ν=0 for any n. Analyzing the saddle point values $\chi(n,0)$ for odd n one can see that n=1 term is the largest.

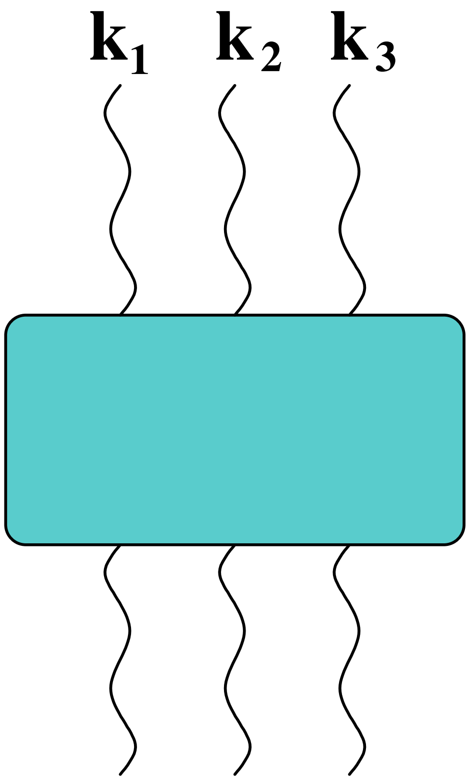
The leading high energy odderon intercept would then be given by

$$\alpha_{odd} - 1 = 2\,\chi(n=1,\nu=0)\frac{\alpha_S N_C}{\pi} = 0$$

in agreement with Bartels, Lipatov and Vacca (BLV).

$\boxed{\text{Odderon amplitude is at most constant with energy!}}$

Connection to BLV Solution



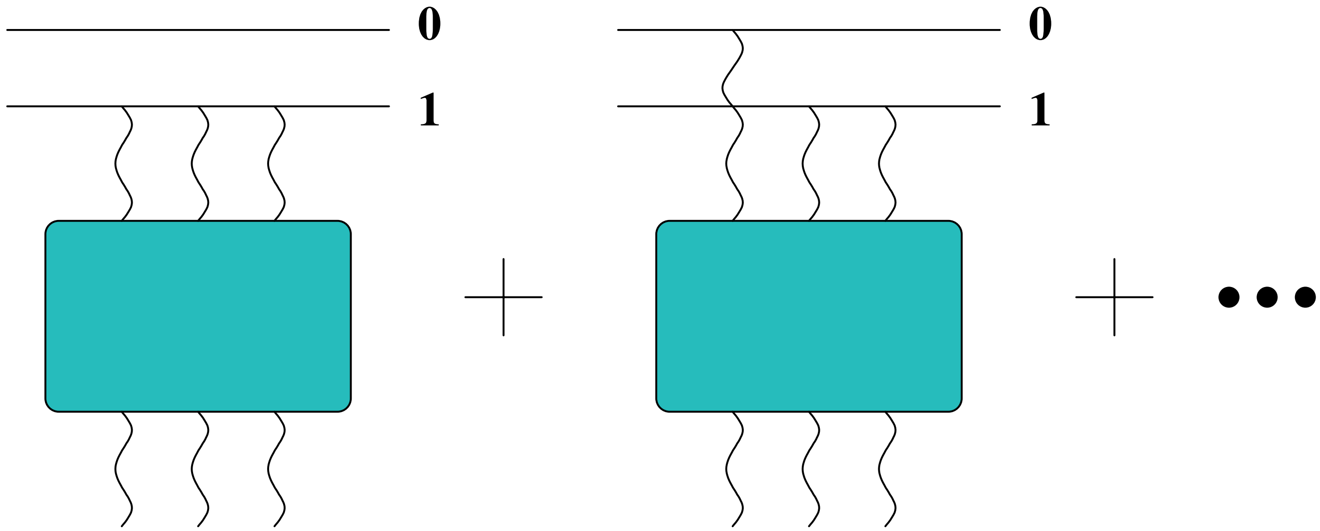
Bartels, Lipatov and Vacca have found the following eigenfunctions of the 3-reggeon BKP equation in momentum space:

$$\Psi^{n,\nu}(k_1,k_2,k_3) = c(n,\nu)\sum_{(123)} \frac{(\underline{k}_1+\underline{k}_2)^2}{\underline{k}_1^2\,\underline{k}_2^2} E^{n,\nu}(\underline{k}_1+\underline{k}_2,\underline{k}_3)$$

where the sum goes over all permutations of indices 1,2,3.

Connection to BLV Solution

Let's see shat the BLV solution translates into for the dipole amplitude in transverse coordinate space. To do that let's connect the BLV amplitude to an incoming dipole 01:



A straightforward calculation shows that BLV eigenfunctions give the dipole amplitude

$$O^{n,\nu}(\underline{x}_0, \underline{x}_1) = 24\, c(n,\nu)\, \pi\, \chi(n,\nu)\, E^{n,\nu}(\underline{x}_0, \underline{x}_1)$$

which is also given by function $E^{n,\nu}(\underline{x}_0, \underline{x}_1)$ up to a constant!

Connection to BLV Solution

⇨ BLV eigenfunctions, when connected to a dipole, give the same eigenfunctions $E^{n,\nu}(\underline{x}_0, \underline{x}_1)$ (with odd n) as found from the dipole evolution equation.

⇨ The corresponding eigenvalues are identical in the BLV and in our cases $\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)$

⇨ This leads to the dominant Odderon intercept $\alpha_{odd} - 1 = 0$

⇨ Our solution is identical to BLV!

⇨ Since we have a complete set of solutions to the dipole equation, we have proven that there is no other odderon solution giving larger intercept than the BLV solution as long as dipole-anything scattering is concerned!

Are we solving the same equation?

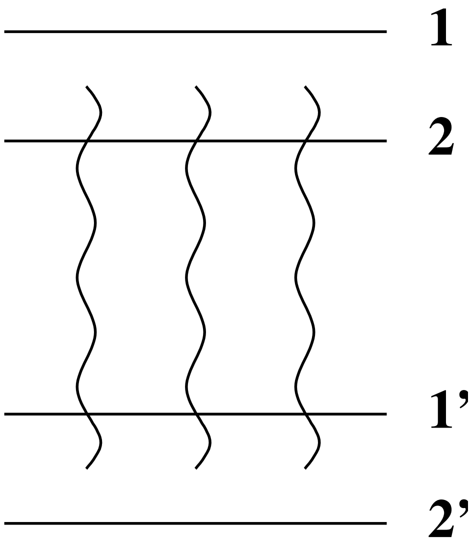
Is there an equivalence between the BKP equation for 3 reggeons and the dipole equation?

Here we are going to show that the equivalence exists, as long as dipole scattering is concerned.

(Many thanks to Al Mueller for discussions which led us to come up with this proof.)

Are we solving the same equation?

Let us start with the same three-gluon initial conditions and see how BKP and dipole equation kernels act on it.



The 3-gluon onium-onium scattering amplitude is equal to

$$O(\rho_1,\rho_2;\rho_{1'},\rho_{2'};Y=0) = c_0\,\alpha_S^3 \ln^3 \left|\frac{\rho_{11'}\,\rho_{22'}}{\rho_{12'}\,\rho_{1'2}}\right|$$

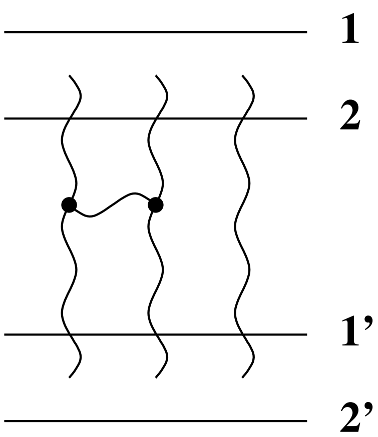
which can be rewritten as

$$O(\rho_1,\rho_2;\rho_{1'},\rho_{2'};Y=0) = \sum_{odd\ n} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0\, C_{n,\nu}\, E^{n,\nu}(\rho_{10},\rho_{20})\, E^{n,\nu}(\rho_{1'0},\rho_{2'0})$$

with coefficients $C_{n,\nu} = c_0\,\alpha_S^3 \dfrac{6}{\pi^2} \dfrac{\nu^2+n^2/4}{\left[\nu^2+(n+1)^2/4\right]\left[\nu^2+(n-1)^2/4\right]}\chi(n,\nu)$

One Rung of BKP

One rung of BKP evolution is shown here (horizontal line is the full BFKL kernel):



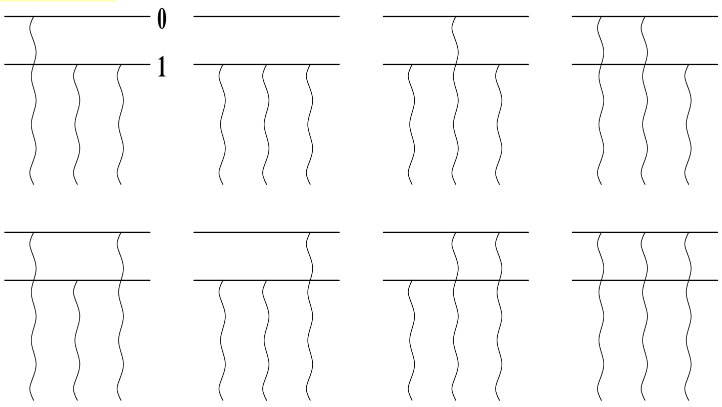
+ **2 more graphs**

BLV showed that BFKL kernel acting on their eigenfunction

$$\Psi^{n,\nu}(k_1,k_2,k_3) = c(n,\nu)\sum_{(123)} \frac{(\underline{k}_1+\underline{k}_2)^2}{\underline{k}_1^2\,\underline{k}_2^2} E^{n,\nu}(\underline{k}_1+\underline{k}_2,\underline{k}_3)$$

gives

$$K_{BKP}\otimes\Psi^{n,\nu} = \frac{2\,\alpha_S\,N_C}{\pi}\,\chi(n,\nu)\,\Psi^{n,\nu}$$



To go to coordinate space we recall that there

$$\Psi^{n,\nu}(k_1,k_2,k_3)\rightarrow E^{n,\nu}(\underline{x}_1,\underline{x}_2)$$

One Rung of BKP

Therefore

$$K^{coord}_{BKP} \otimes E^{n,\nu}(\underline{x}_1, \underline{x}_2) = \frac{2\alpha_S N_C}{\pi} \chi(n,\nu) E^{n,\nu}(\underline{x}_1, \underline{x}_2)$$

Since

$$O(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y = 0) = \sum_{odd\, n} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0 \, C_{n,\nu} \, E^{n,\nu}(\rho_{10}, \rho_{20}) \, E^{n,\nu}(\rho_{1'0}, \rho_{2'0})$$

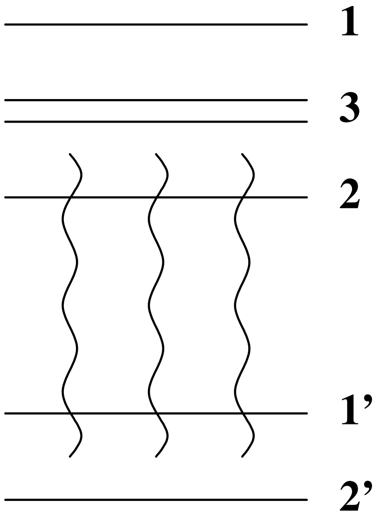
than

$$K^{coord}_{BKP} \otimes O(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y = 0) =$$

$$= \frac{2\alpha_S N_C}{\pi} \sum_{odd\, n} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0 \, \chi(n,\nu) \, C_{n,\nu} \, E^{n,\nu}(\rho_{10}, \rho_{20}) \, E^{n,\nu}(\rho_{1'0}, \rho_{2'0})$$

One Rung of Dipole Evolution

In the rest frame of one of the onia a rung of dipole evolution looks like



Since, as we showed above

$$K_{dipole} \otimes E^{n,\nu}(\underline{x}_1, \underline{x}_2) = \frac{2\alpha_S N_C}{\pi} \chi(n,\nu) E^{n,\nu}(\underline{x}_1, \underline{x}_2)$$

than

$$K_{dipole} \otimes O(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y = 0) =$$
$$= \frac{2\alpha_S N_C}{\pi} \sum_{odd\ n} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0 \, \chi(n,\nu) C_{n,\nu} E^{n,\nu}(\rho_{10}, \rho_{20}) E^{n,\nu}(\rho_{1'0}, \rho_{2'0})$$

⇨ Higher order iterations will be the same by induction.

⇨ BKP and dipole equations are identical! (for dipole scattering)

Linear Evolution Summary

⇨ We have shown that for dipole-anything scattering 3-reggeon BKP equation is equivalent to the dipole evolution equation.

⇨ The dipole evolution equation has a well-known complete set of eigenfunctions $E^{n,\nu}(\underline{x}_0, \underline{x}_1)$ and eigenvalues. C-odd initial conditions single out the ones with odd n.

$$\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)$$

⇨ We have therefore constructed a <u>complete set of solutions</u> for the odderon exchange amplitude, which turned out to be equivalent to the BLV solution.

⇨ The leading Odderon intercept is $\alpha_{odd} - 1 = 0$

Saturation/Color Glass Effects

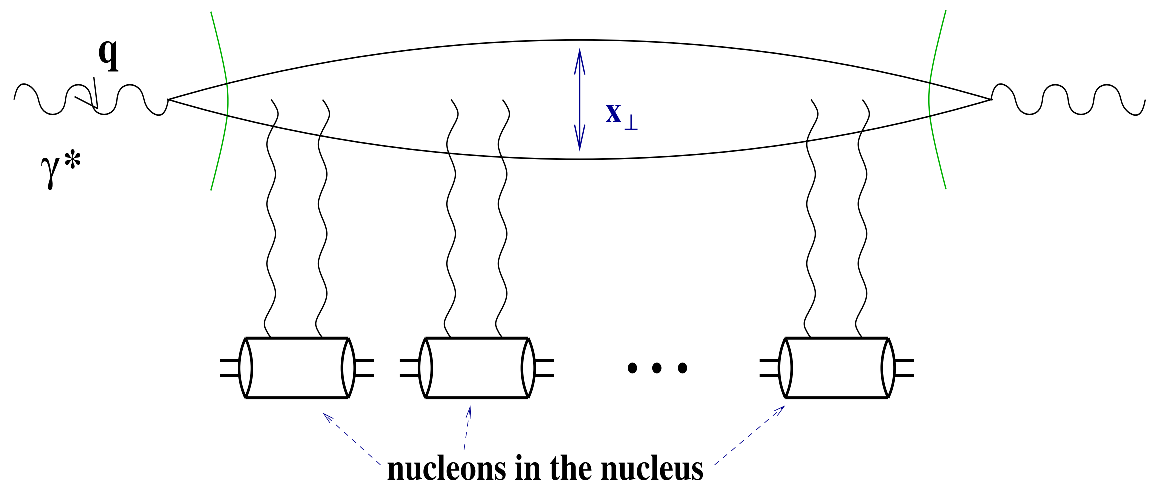
At very high energies the gluonic fields become strong leading to parton saturation.

Saturation is crucial for unitarization of total DIS cross sections (at fixed impact parameter) mediated by the BFKL pomeron exchange. Saturation effects slow down the corresponding growth of the cross sections from power-of-energy ($\sim s^{\Delta}$) to a constant.

With zero intercept odderon exchange amplitude is in no danger of violating unitarity bound. Nevertheless, the saturation effects can give an order 1 correction to the odderon exchange amplitude.

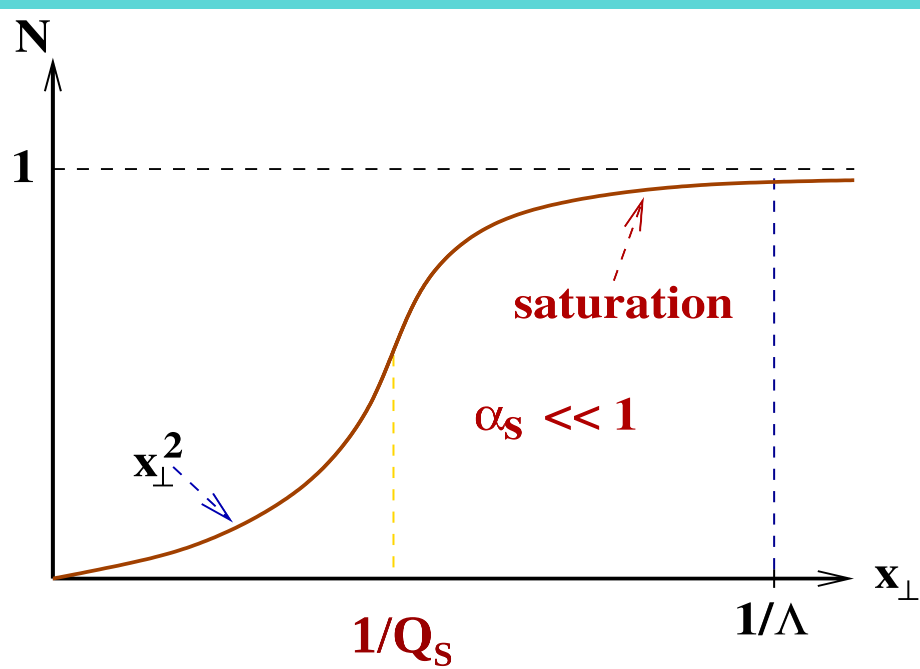
Saturation effects for C-even Amplitude

In the rest frame of the target the DIS process is shown on the right, and it factorizes into



$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = \Phi^{\gamma^* \to q\bar{q}} \otimes N(x_\perp, Y = \ln(1/x_{Bj}))$$

with rapidity Y=ln(1/x)



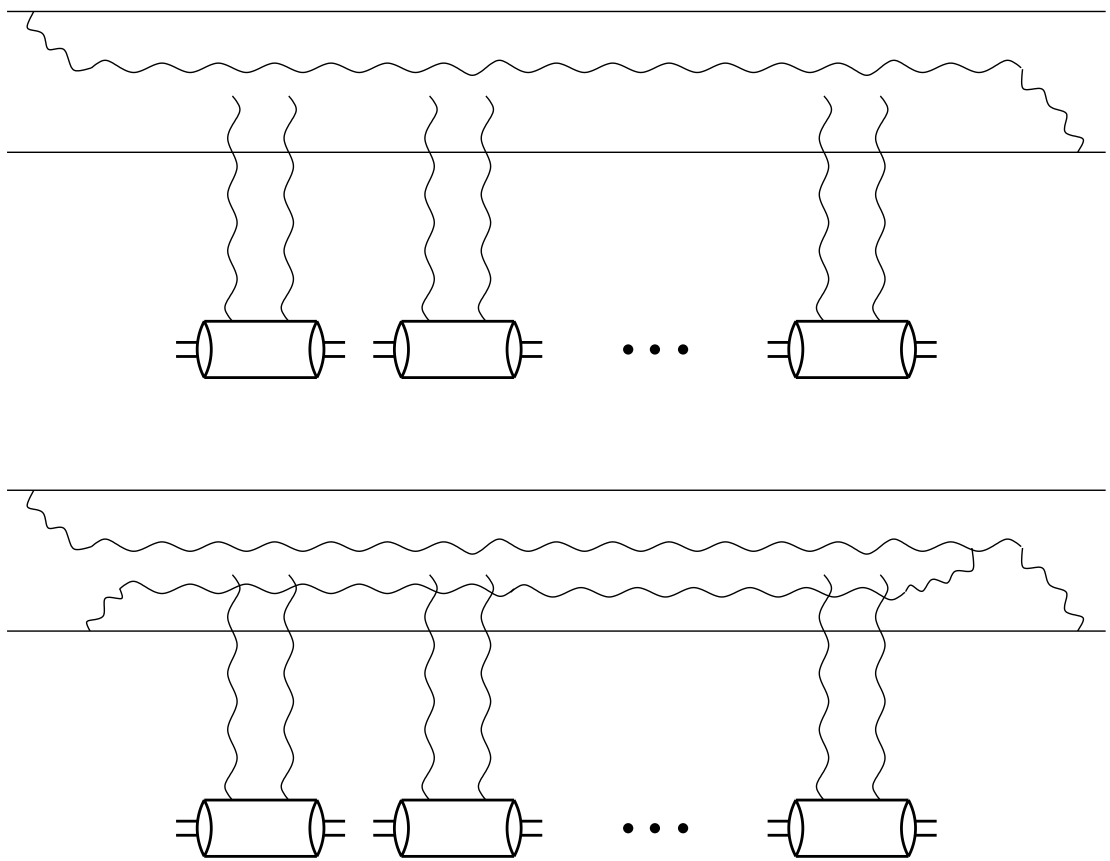
The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4} \ln \frac{1}{x_\perp \Lambda}\right]$$

A.H. Mueller, '90

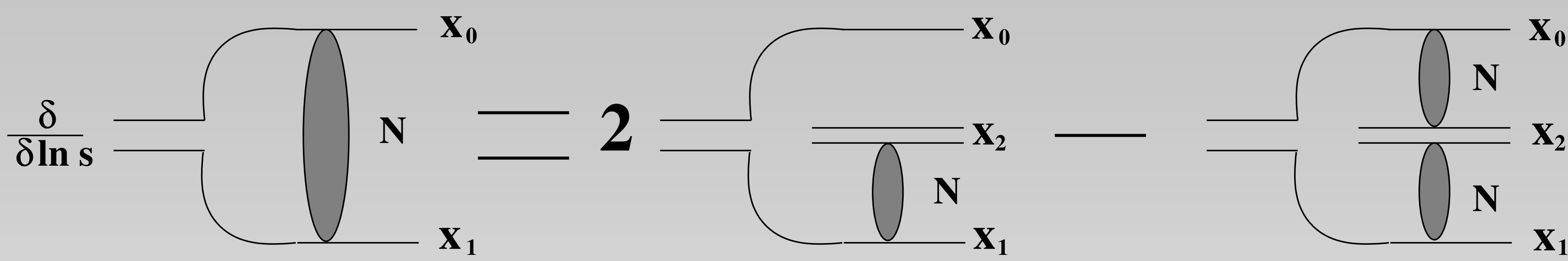
Quantum Evolution

As energy increases the higher Fock states including gluons on top of the quark-antiquark pair become important. They generate a cascade of gluons.



These extra gluons bring in powers of $\alpha_S \ln s$, such that when $\alpha_S << 1$ and $\ln s >> 1$ this parameter is $\alpha_S \ln s \sim 1$.

Nonlinear Evolution Equation



Defining rapidity Y=ln s we can resum the dipole cascade

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2}\int d^2x_2 \left[\frac{x_{01}^2}{x_{02}^2\, x_{12}^2} - 2\pi\,\delta^2(\underline{x}_{01}-\underline{x}_{02})\ln\left(\frac{x_{01}}{\rho}\right)\right] N(x_{02},Y) - \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}\, N(x_{02},Y)\, N(x_{12},Y)$$

Yu. K., '99, large N_C QCD
I. Balitsky, '96, HE effective lagrangian

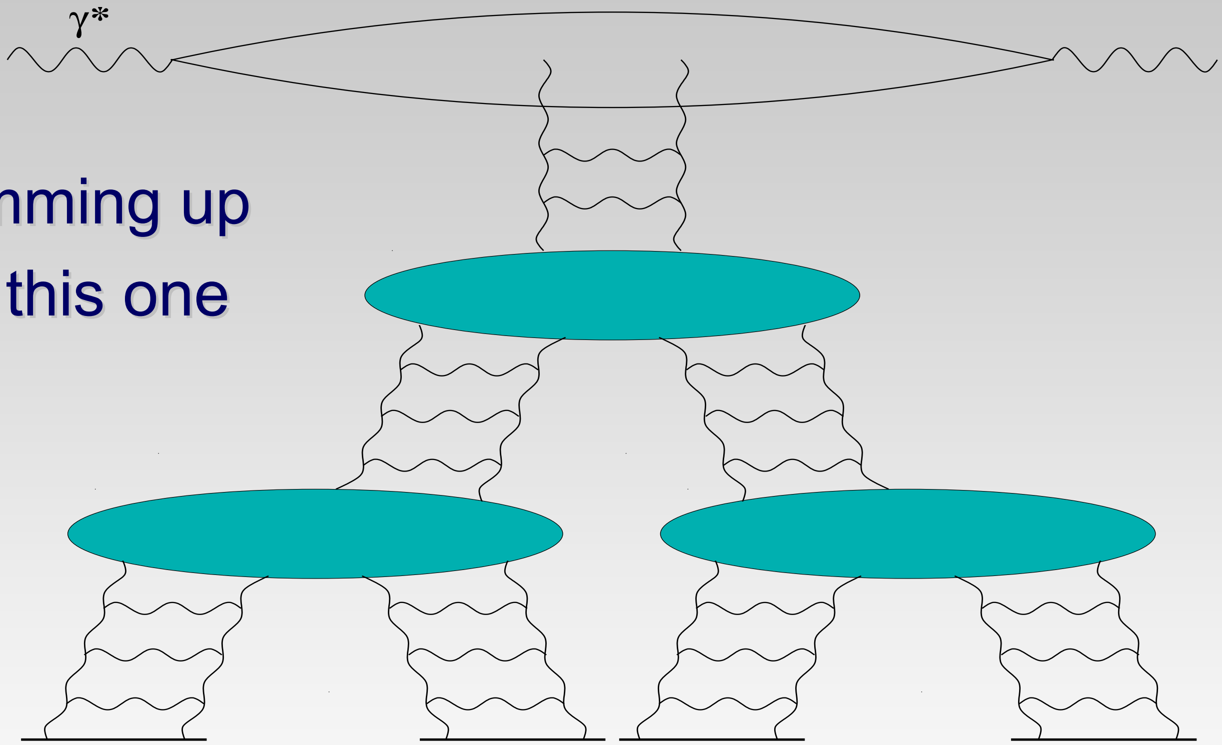
$$N(x_\perp, Y=0) = 1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4}\ln\frac{1}{x_\perp \Lambda}\right]$$

initial condition

⇨ Linear part is BFKL, quadratic term brings in damping

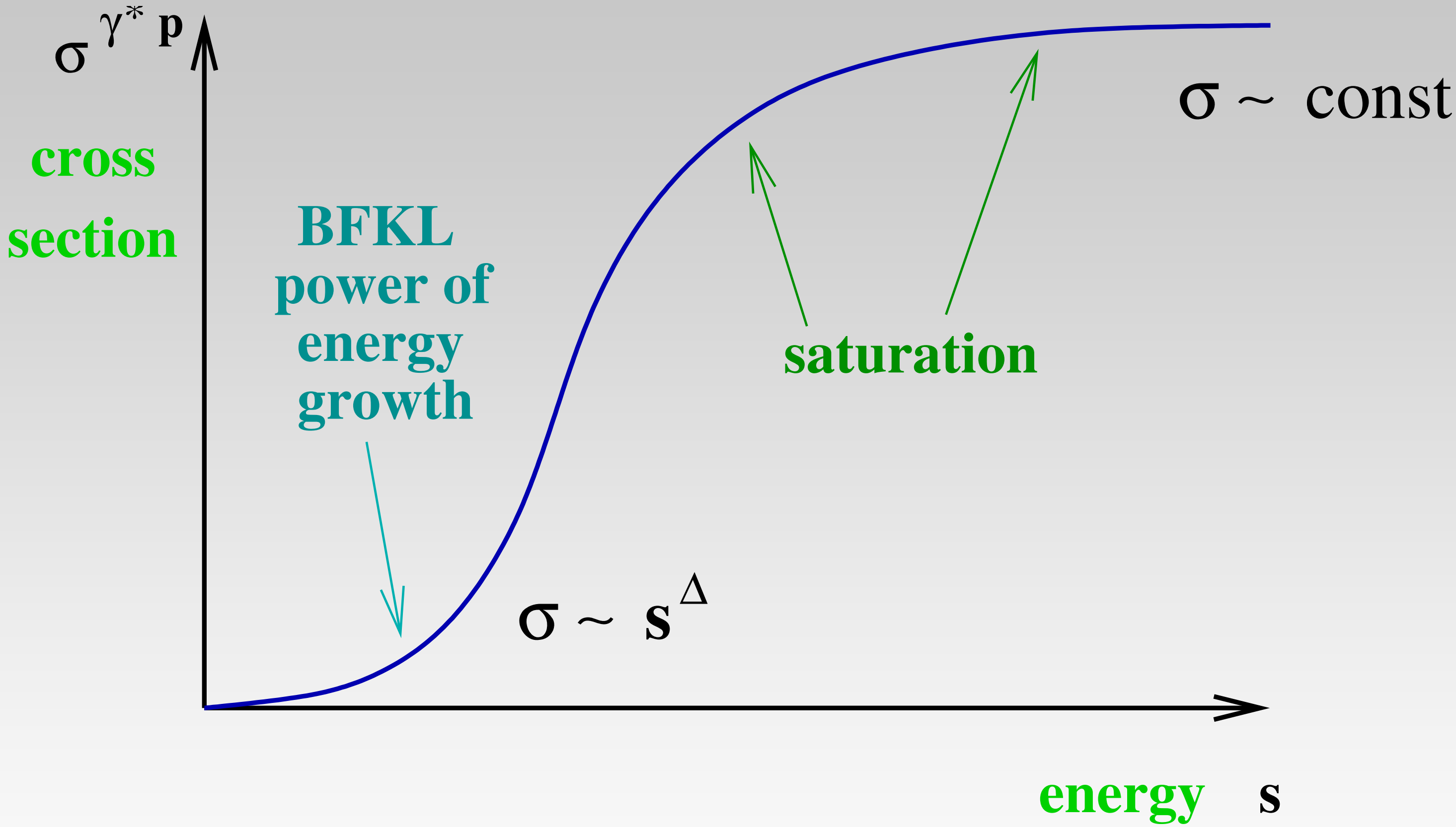
Nonlinear Equation

In the traditional language this corresponds to summing up “fan” diagrams like this one



Gluon recombination in the nuclear wave function corresponds to splitting in the projectile wave function.

Nonlinear Equation: Saturation

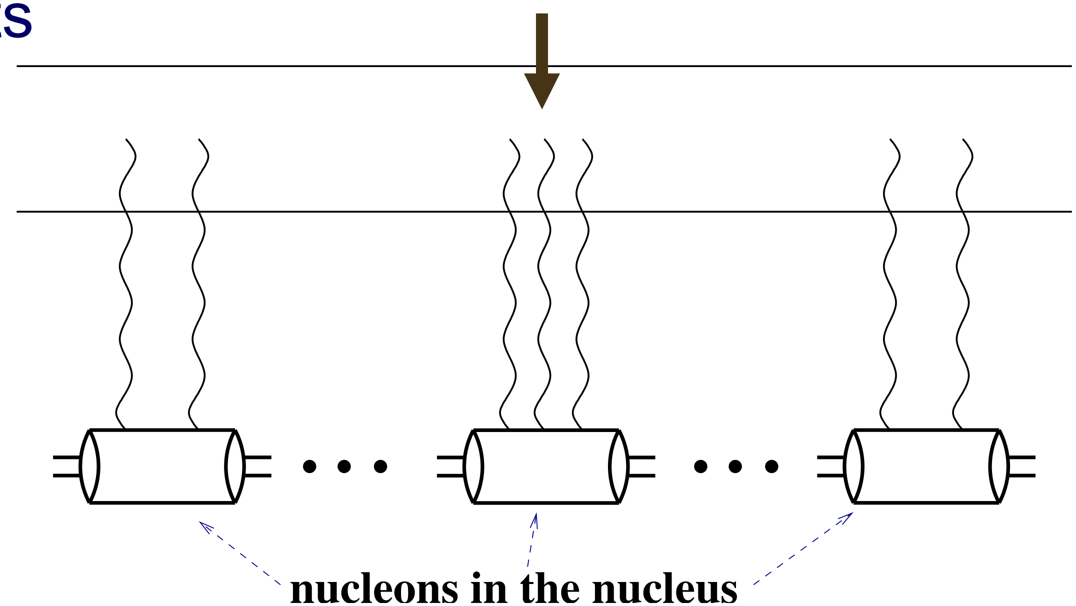


Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

Including Saturation Effects in the Odderon Amplitude

To include saturation effects in odderon evolution one has to first include multiple rescatterings a la Glauber-Mueller / McLerran-Venugopalan:

LO Odderon

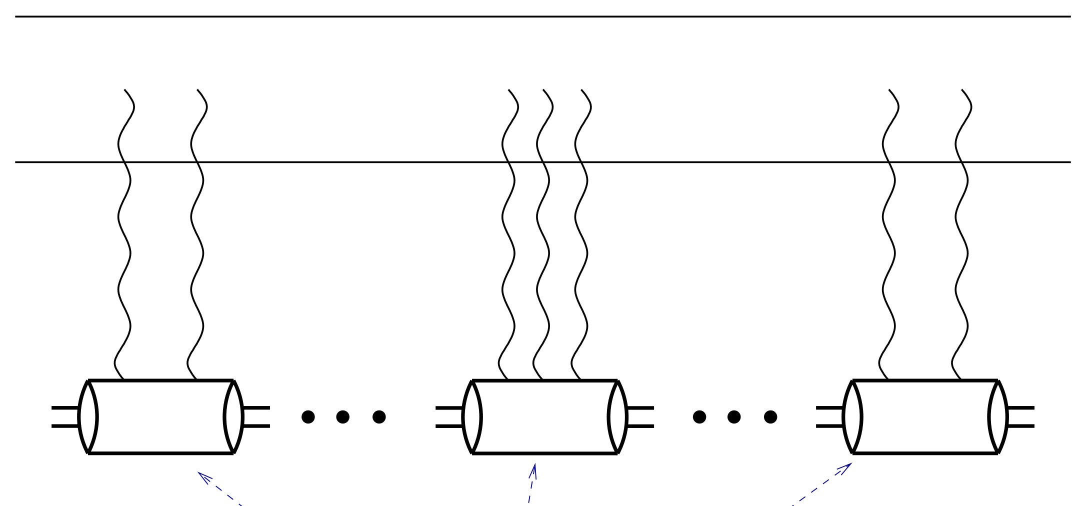


For a nuclear target, two-gluon multiple rescatterings bring in powers of $\alpha_s^2 \, A^{1/3} \sim 1$, so that single odderon exchange

makes the amplitude to be of the order $\alpha_s^3 \, A^{1/3} \sim \alpha_s << 1$

Including Saturation Effects

Resummation of multiple rescatterings gives initial conditions for the evolution which we will construct next:



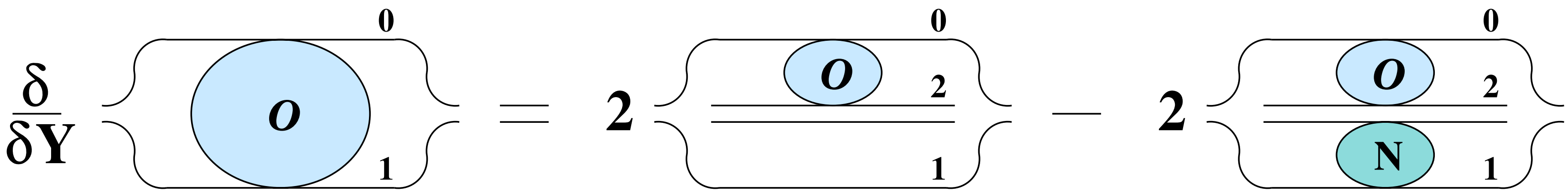
(cf. Jeon and Venugopalan, '05)

$$O(\underline{x}_0, \underline{x}_1, Y=0) =$$

$$= \underbrace{c_0 \alpha_S^3 \rho \int_{S_\perp} d^2 b\, T(b) \ln^3 \left(\frac{|\underline{B} - \underline{b} + (1/2)\underline{x}_{01}|}{|\underline{B} - \underline{b} - (1/2)\underline{x}_{01}|} \right)}_{odderon\ \ exchange} \underbrace{e^{-x_{01}^2 Q_S^2/4}}_{\substack{multiple \\ rescatterings}}$$

Odderon Evolution in Color Glass

Now one can write down a nonlinear evolution equation for the odderon amplitude, including the effects of multiple pomeron exchanges in the color glass condensate:



$$\frac{\partial O(\underline{x}_0,\underline{x}_1,Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}\left[O(\underline{x}_0,\underline{x}_2,Y) + O(\underline{x}_2,\underline{x}_1,Y) - O(\underline{x}_0,\underline{x}_1,Y)\right]$$
$$- \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}\left[O(\underline{x}_0,\underline{x}_2,Y)\, N(\underline{x}_2,\underline{x}_1,Y) + N(\underline{x}_0,\underline{x}_2,Y)\, O(\underline{x}_2,\underline{x}_1,Y)\right]$$

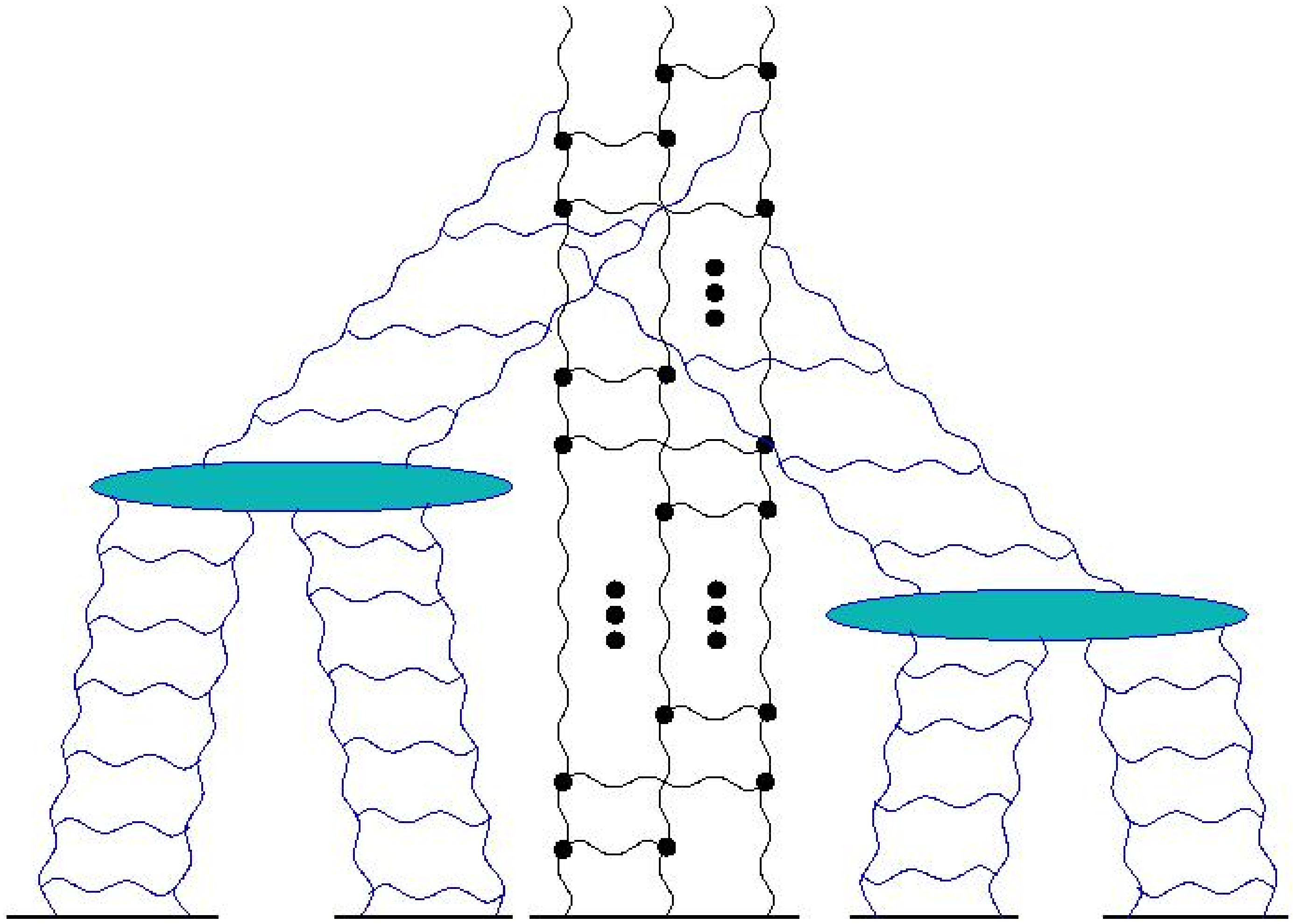
(cf. evolution equation for reggeon by K. Itakura, Yu. K., L. McLerran, D. Teaney, ’03 and generalization of this equation by Hatta, Iancu, Itakura, McLerran, ‘05)

Here N is the C-even (forward) dipole amplitude on the target. N varies from ~0 at low energies to 1 at high energies.

⇨ Saturation effects only <u>decrease</u> the Odderon intercept!

Feynman Diagram Analogy

In the traditional Feynman diagram language we are resumming graphs like



Saturation Summary

⇨ We have constructed an evolution equation for the odderon amplitude in the saturation / Color Glass regime.

⇨ The equation appears to lead to a decrease of the odderon intercept at very high energy. That means that the intercept will go from 0 to negative as energy increases, making the odderon amplitude a <u>decreasing</u> function of energy.

⇨ Saturation has (probably) been observed at HERA. Does it mean that we should expect the odderon exchange amplitude to be a decreasing function of energy at low Q^2 in HERA kinematics? Any other phenomenological predictions? Can saturation be the reason why they have not found odderon yet?

Conclusions

⇨ We have found the most general solution for the Odderon exchange amplitude for a dipole scattering on any target and showed that it is equivalent to the BLV solution. We have shown that the leading high energy odderon intercept is given by

$$\boxed{\alpha_{odd} - 1 = 0}$$

in agreement with the results of Bartels, Lipatov and Vacca.

⇨ We have shown that in Mueller's dipole model both the BFKL Pomeron and the Odderon are described by the same evolution equation with C-even and C-odd init. conditions correspondingly.

⇨ We have written an evolution equation for the Odderon in the Color Glass regime and showed that saturation can only reduce the Odderon intercept at high energies.